Applications

Student Name	
Department	
Section No.	

Exercises 1

Prove that

$$\boxed{\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1}$$

Exercises 2

If $z = x + jy$, find the equation of the locus $\arg(z^2) = \dfrac{\pi}{4}$

Exercises 3

1-Expand $\sin 4\theta$ in powers of $\sin\theta$ and $\cos\theta$.

2- Express $\cos^4\theta$ in terms of cosines of multiples of θ.

3- If $z = x + jy$, find the equations of the two loci defined by

(a) $|z - 4| = 3$ (b) $\arg(z + 2) = \dfrac{\pi}{6}$

Exercises 4

Show that $u(x, y) = x^3y - y^3x$ is an harmonic function and find the function $v(x, y)$ that ensures that $f(z) = u(x, u) + jv(x, y)$ is analytic. That is, find the function $v(x, y)$ that is conjugate to $u(x, y)$.

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Exercises 5 (Harmonic functions)

Are the following functions harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

1. $u = e^{-x} \sin 2y$
2. $u = xy$
3. $v = xy$
4. $v = -y/(x^2 + y^2)$
5. $u = \ln |z|$
6. $v = \ln |z|$
7. $u = x^3 - 3xy^2$
8. $u = 1/(x^2 + y^2)$
9. $v = (x^2 - y^2)^2$

Exercises 6

Determine a, b, c such that the given functions are harmonic and find a harmonic conjugate.

1- $U = ax^2 + y^2$

2. $u = e^{3x} \cos ay$

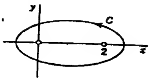
3. $u = \sin x \cosh cy$

4. $u = ax^3 + 5y^3$

Exercises 7

Evaluate

$\oint_C \frac{7z - 6}{z^2 - 2z}\, dz$, C as shown



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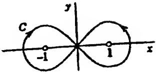
xercises 8

valuate

$$\oint_C \frac{dz}{z^2 - 1}\text{, } C \text{ as shown}$$



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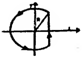
Exercises 9

(a) **Evaluate** $\oint_C \frac{z}{(z-1)(z+2i)}dz$ **around** $C:|z|=4$.

(b) Using Bromwich contour



To find inverse Laplace transform of

$$F(s) = \frac{1}{(s-1)(s-2)}$$

xercises 1

xpand $\dfrac{e^{3z}}{(z-2)^4}$ in a Laurent series about the point $z = 2$ and etermine the nature of the singularity at $z = 2$.

Exercises 11

1. Find the Laurent series about the point indicated of each of the following.

(a) $\frac{1}{z}\sin\left(\frac{1}{z}\right)$ about the point $z = 0$

(b) $\frac{1}{2z-3}$ about the point $z = 3/2$

(c) $\frac{z}{(z-2)(z-3)}$ about the point $z = 3$.

2. Find the Laurent series of $\frac{z-1}{(z+2)(z+5)}$ that is valid for

(a) $2 < |z| < 5$

(b) $|z| > 5$

(c) $|z| < 2$.

ercises 12

valuate $\displaystyle\int_0^{2\pi} \frac{1}{4\cos\theta - 5}\, d\theta.$

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Exercises 13

Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^4}\,dx$.

Exercises 14

1 Map the following points in the z-plane onto the w-plane under the transformation $w = f(z)$.

(a) $z = 3 + j2$; $w = 2z - j6$

(b) $z = -2 + j$; $w = 4 + jz$

(c) $z = j(1 - j)$; $w = (2 + j)z - 1$

(d) $z = j - 2$; $w = (1 - j)(z + 3)$.

2 Map the straight line joining A $(2 - j)$ and B $(4 - j3)$ in the z-plane ont the w-plane using the transformation $w = (1 + j2)z + 1 - j3$. State th magnification, rotation and translation involved.

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Exercises 15

Evaluate the integrations using Gamma and beta function

(i) $\int_0^{\infty} \sqrt{x}\; e^{-x^3}\, dx$ (ii) $\int_0^1 x^m (\ln x)^n\, dx$

(iii) $\int_0^1 \sqrt{\frac{1}{x} - 1}\; dx$

ercises 16

aluate the integrations using Gamma and Beta functions

$\int_0^\infty (1+\sqrt{x})^2 \; e^{-x} \; dx$ (ii) $\int_{-\infty}^{\infty} e^{(3x-e^x)} \; dx$

i) $\int_0^1 (x^3+\sqrt{x})^2 \; \sqrt{1-x} \; dx$

$\int_2^5 (x-2)^3 \; (5-x)^{\frac{1}{3}} dx$

Exercises 17

(a)Prove that

(i) $$e^{\alpha \sin\theta} = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos 2n\theta + 2i\sum_{n=0}^{\infty} J_{2n+1}(x)\sin(2n+1)\theta$$

(ii) $$1 = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)$$

(iii) $$x = 2\sum_{n=0}^{\infty}(2n+1)\, J_{2n+1}(x)$$

prove that cos(xsin

)+...θ) $+2\ J_6 \cos(6\theta) + 2J_4 \cos(4\theta)=J_0+2\ J_2\ \cos(2\theta$ sin(xsin

)+...θ) $+2\ J_5 \sin(5\theta) + 2J_3 \sin(3\theta)=2\ J_1 \sin(\theta$

Let sets A , B and C be fuzzy sets defined on real numbers by membership functions

$$\mathfrak{A}_A(x)=\frac{x}{x+1} \quad , \mathfrak{A}_B(x)=\frac{1}{x^2+10} \; , \mathfrak{A}_C(x)=\frac{1}{10^x}$$

Determine mathematical membership functions graphs of the followings

$a) A \cup B$, $B \cap C$, $b) A \cup B \cup C$, $A \cap B \cap C$

$c) A \cap C$, $B \cup C$ $d) A \cap B$, $A \cup B$

Show the two fuzzy sets satisfy the De Morgan s Law ,

$$\mathfrak{A}_A = \frac{1}{1+(x-10)} \quad , \quad \mathfrak{A}_B(x) = \frac{1}{1+x^2}$$

Evaluate $\oint_c \frac{e^z}{z^2+1}dz$, $\oint_c \frac{\cos z \, dz}{z^2(z+2)}$, $\oint_c \frac{dz}{z^2(z+4)}$

where C is the circle $|z-1|=2$

Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2$ is a harmonic function and find the corresponding analytic function $f(z) = u + iv$

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If in the function $f(z)=u+iv$, we take z in polar form, namely

$z=re^{i\theta}=r(\cos\theta+i\sin\theta)$

Show that the Cauchy - Riemann equations become

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}\ ,\quad \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Example

Show that $f(z)=u(r,\theta)+iv(r,\theta)$ is analutic, then

$r^2 u_{rr}+ru_r+u_{\theta\theta}=0$

Solution

$u_r=\frac{1}{r}v_\theta$ $\qquad\qquad v_r=-\frac{1}{r}u_\theta$ (C-R)

iff. w.r.to r $\quad u_r=\frac{1}{r}v_\theta$

$u_{rr}=r^{-1}v_{\theta r}-r^{-2}v_\theta$

$r^2u_{rr}=rv_{\theta r}-v_\theta$

Form

$u_\theta=-rv_r$ $\qquad$ diff – w.r.to $.\theta$

$u_{\theta\theta}=-rv_{r\theta}$

From (1),(2)

$r^2u_{rr}+ru_r+u_{\theta\theta}=rv_{\theta r}-v_\theta+v_\theta-rv_{r\theta}-rv_{r\theta}$

$r^2u_{rr}+ru_r+u_{\theta\theta}=0$

Example

If $f(z)=u(x,y)+iv(x,y)$ is analytic in a region R, prove that the one parameter families of cerues

$U(x,y)=C1$ and $v(x,y)=C2$ are Orthogonal families.

Solution :

$u(x, y) = c_1$ **then** $du = u_x dx + u_y dy = 0$

$$\therefore \frac{dy}{dx} = -\frac{u_x}{u_y} \qquad (1)$$

Also $V(x, y) = c_2$ **then** $dv = v_x dx + v_y dy = 0$

$$\frac{dy}{dx} = -\frac{v_x}{v_y} \qquad (2)$$

By Cachy – **Rieman equations, we have the product of the slopes** at a fixed point

$$\left(-\frac{U_x}{U_y}\right)\left(-\frac{V_x}{V_y}\right) = -1$$

So that any two **members of the respective families are Orthogonal**